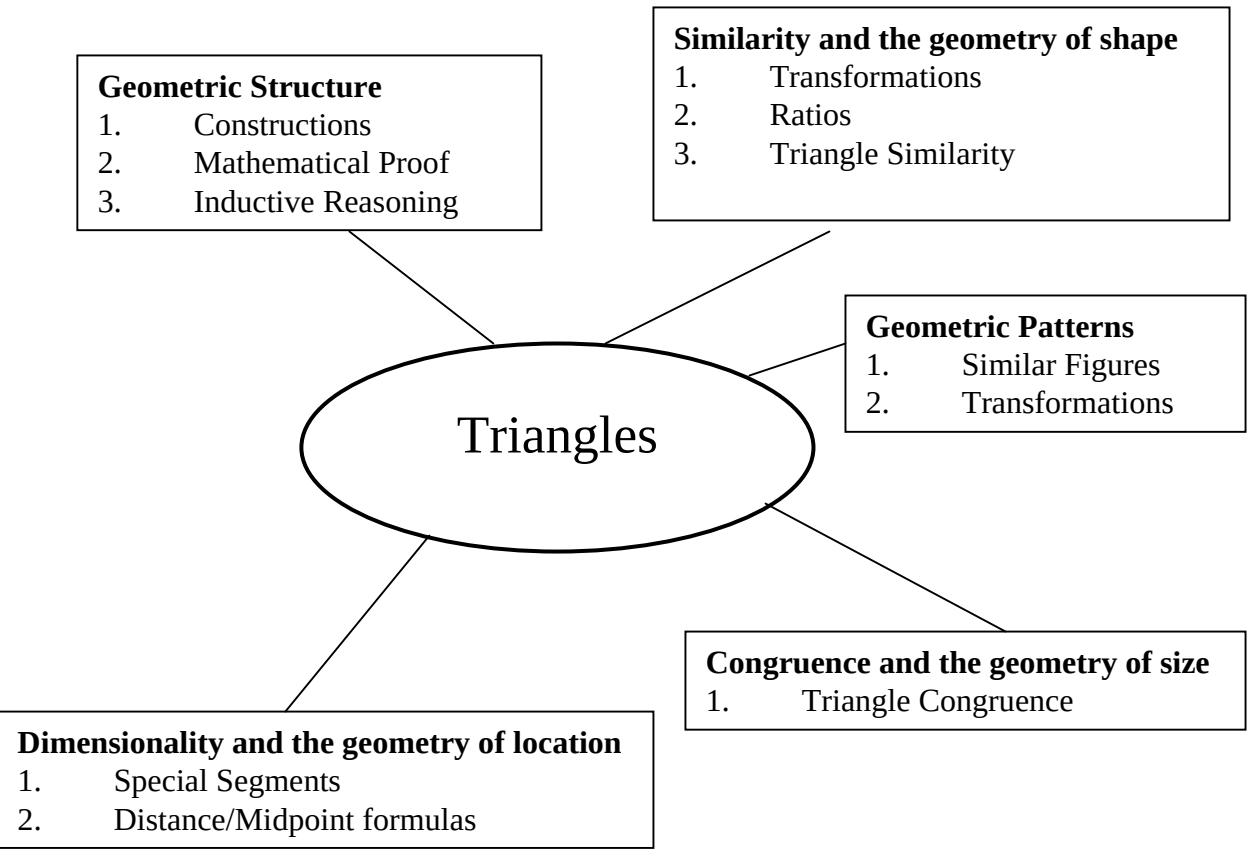
Unit Planner

Unit Title: Triangles

Conceptual Lens: Transformations

Length: 2 weeks



Unit Overview

How are triangles used in the construction of buildings? How does changing one angle of a triangle affect the remaining angles? What about the sides? This unit will cover classifying triangles, similarity in triangles, using patterns, and special segments of triangles.

Designer(s): Ghere, David

Grade Level: Geometry

Enduring Understandings

(generalizations)

Stnd.

1. Properties of shapes establish congruence or similarity. Measure of sides and angles are used to verify congruence or similarity. Congruence and similarity allow you to link real world objects to geometric concepts.

2. Proportions may be used to describe relationships between similar shapes. Ratios allow proportions to be set up. Algebra skills give ways to solve the proportions for unknown values.

3. Special segments of triangles show different relations among angles and midpoints. Lines drawn from the angles and midpoints of a triangle create segments that intersect. The intersections can show relationships between circles and triangles.

Guiding Questions

Stnd.

1. Is it possible to find the missing angle of a triangle if you know only two of the angles in it?
2. How could you determine which is the *longest* side of a triangle if you only know the 3 angles? *Shortest*, *Middle*?
3. Is it possible to determine if two triangles are congruent if you aren't given enough information to calculate all sides or angles of the triangle?
4. What does it take for 2 triangles to be similar to each other?
5. Why are Algebra skills involving ratios and proportions important in solving similar triangle problems?
6. How can special lines drawn inside triangles make solving problems possible?
7. How can the midpoint formula be used to draw the midsegment for any triangle?
8. Can the coordinate system play a role in finding the midsegment given in the previous question?

Critical Content and Skills:

AC=Assessment Code: Q – Quizzes P – Prompts T – Tests O – Observations WS – Work Samples D - Dialogues

Students will know…	Stnd.	AC		Stnd.	AC
1. how to classify triangles according to their properties.	Gb2A	WS	5. how ratios are used to find missing information	Gf2	O, Q
2. how to find unknown angles of triangles.	Ge2B	WS	6. how to identify special segments and use them to solve problems involving triangles.	Gd2B	D
3. how to determine the longest, middle, or shortest side of a triangle from the angles that are given.	Ge2B	Q	7. how triangles can be mapped onto coordinate systems.	Gd2B	WS
4. how logic can prove that two triangles are congruent to each other. (SSS, SAS, ASA, AAS)	Ge3A, Ge3B	P, WS, Q	8. how the coordinate system is used in applying transformations.	Gd2B	D

Key Skills…	Stnd.	AC		Stnd.	AC
1. Grouping shapes by their properties.	Gb2A	WS	5. Constructions.	Gb2A	O
2. Deductive Reasoning.	Gb3B	D	6. Plotting Points	Gd2C	WS
3. Application of Theorems, Postulates and Definitions.	Gb1A	WS	7. Solving Equations.		
4. Setting up and solving a proportion.	Gf2	Q			

Task Planner

What: Investigate…the measures of the angles of a triangle.

Why: In order to understand that… the sum of all angles in a triangle is 180 degrees.

How: Paper cutting activity from Heath textbook pp. 169-170.

1. The students will draw a triangle on a sheet of paper, and then cut it out.
2. They will then label the angles on the triangle.
3. After this they will tear off the three angles.
4. Then have them put the angles together connecting their vertices. (This shows the *Triangle Sum Theorem*.)
5. Have the students write an essay on their findings. Be sure to have them answer these questions…
    a. What type of angle was formed by all three when placed together?
    b. Would this work for all triangles?
    c. How could you show that it would work for all triangles?

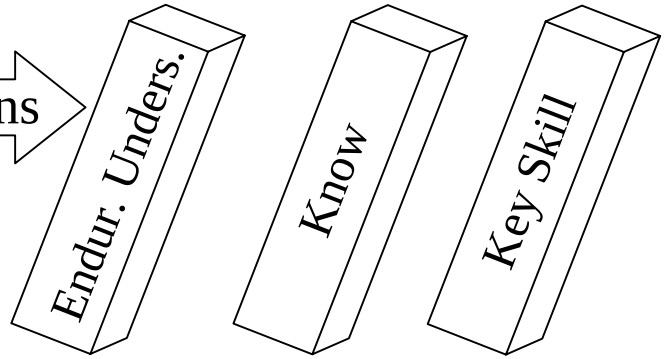
Mathematics Performance Assessment Rubric

Part a) Correct Solution YES NO

Criteria	4	3	2	1
Part b) **Conceptual Knowledge**	**Attribute(s) of concept(s)** Correctly identifies attributes of the problem, which leads to correct inferences. **Inferences** Combines the critical attributes of the problem in order to describe correctly the mathematical relationship(s) inherent in the problem.	**Attribute(s) of concept(s)** Correctly identifies attributes of the problem, which leads to correct inferences. **Inferences** Combines the critical attributes of the problem, which leads to a partial identification of the mathematical relationship(s) inherent in the problem.	**Attribute(s) of concept(s)** Identifies some of the attributes of the problem, which leads to partially correct inferences. **Inferences** Combines the identified attributes of the problem, which leads to a partial identification of the mathematical relationship(s) inherent in the problem.	**Attribute(s) of concept(s)** Lacks identification of any of the critical attributes of the problem. **Inferences** Combines few of the attributes of the problem which leads to an incomplete identification of the mathematical relationship(s) inherent in the problem.
Part c) **Procedural Knowledge**	**Appropriate strategy** Selects and implements an appropriate strategy. **Representational form** Uses appropriate representation to connect the procedure to the concept of the problem. **Algorithmic competency** Correctly implements procedure to arrive at a correct solution.	**Appropriate strategy** Selects and implements an appropriate strategy. **Representational form** Uses appropriate representation to connect the procedure to the concept of the problem. **Algorithmic competency** Implements selected procedure but arrives at an incorrect solution.	**Appropriate strategy** Selects and implements an appropriate strategy. **Representational form** Uses inconsistent or insufficient representation for the selected solution strategy. **Algorithmic competency** Implements selected procedure but arrives at an incorrect or correct solution. (See Part a above)	**Appropriate strategy** Selects and implements an inappropriate strategy. **Representational form** Uses incorrect representations. **Algorithmic competency** Makes significant errors.
Part d) **Communication**	**Justification** Fully answers the question of "why" for the strategy selection; explains procedure; and/or evaluates reasonableness of solution. **Terminology** Uses appropriate terminology and notation.	**Justification** Fully answers the question of "why" for the strategy selection; explains procedure; and/or evaluates reasonableness of solution. **Terminology** Uses some appropriate terminology or notation.	**Justification** Incompletely answers the question of "why" for the strategy selection; explains procedure; and/or evaluates reasonableness of solution. **Terminology** Uses some appropriate terminology or notation.	**Justification** Provides very little or no explanation of what was done and why. **Terminology** Uses limited or inappropriate terminology or notation.

Suggested Instructional Activities

Correlations



1. TEXTEAMS High School Geometry
    1. 4.1 Triangles Tell it
        a. Teacher Notes: pp. 62-63
        b. Student Activity: pp. 67-68
    2. 4.2 Midpoint Triangle
        a. Teacher Notes: pp. 64-66
        b. Student Activity: pp. 69-70

1.1	1	1, 4	1, 2, 5
1.2	2	6	4, 7

Unit Resources

1. TEXTEAMS High School Geometry

Teacher Notes

Conjectures Activity 1: Triangles Tell it All

1. **Construct a large scalene, acute triangle on an index card.**
   Have participants construct the triangle using a straight edge and whatever other tools they choose. Be sure the triangle covers most of the index card. Model on the overhead projector.

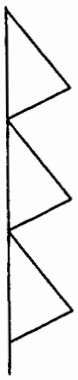
2. **Cut out the triangle.**
   Have participants cut out their triangle.

3. **Verify that your triangle is scalene, acute.**
   Have participants share their methods of verification. They may use a protractor to measure angles, a compass to compare sides, a ruler, paper folding, patty paper or other methods.

4. **Color each angle of the triangle a different color.**
   Have participants use markers to color each angle of their triangle a different color. An alternate method would be to number the angles. Model on the overhead using a triangle cut from transparency film.

5. **On a blank sheet of paper, construct a segment close to and parallel to the long edge. Make the segment as long as possible.**
   After participants construct their segment ask how they might verify that the segment is parallel to the edge of the paper.

6. **At the top of the segment match one edge of the triangle and trace the triangle.**
   This should look like the top triangle to the right.



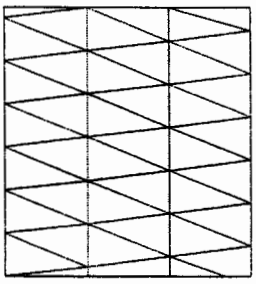
7. **Translate your triangle down the segment until the top vertex now rests on the bottom vertex. Trace the triangle. Color the angles the same as the original triangle.**
   Be sure to emphasize the vocabulary: translate, segment, vertex. Participants should color the angles to match their original triangle.

8. **Repeat step 7 until you run out of room. (You should have something that looks like saw teeth at right.)**
   Model on the overhead then circulate to be sure no one is having difficulty.

9. **Take the triangle and place it on top of the first traced triangle.**
   Model on the overhead projector.

10. **Rotate the triangle 180^0 about the midpoint of either of the 2 sides that are not on the original segment. Trace the triangle and color the angles the same as the original triangle.**
    Emphasize the terms midpoint and rotation. Discuss the degrees of rotation (180). Ask participants how they located the midpoint.

11. Repeat step 10 on each triangle until you have filled the page.
Participants' papers may resemble the diagram below.



12. What geometric relationships can you discover in this picture?
Have participants take two minutes to individually record observations. After they record their observations they should then share within their group. Have each group list their observations on chart paper and post their lists. Have participants do a "gallery walk" comparing their list to others. Did other groups make observations that you did not?

Possible observations may include:
- The sum of the measures of the angles of a triangle is 180 degrees.
- Vertical angles are congruent.
- The measure of an exterior angle of a triangle is equal to the sum of the remote interior angles.
- Opposite angles of a parallelogram are congruent.

13. Select two relationships and justify/prove that the relationships are true. Explain your thinking.
Possible justifications may include:
- The sum of the measures of the angles of a triangle is 180 degrees because when each of the three angles is adjacent to each other a pair of opposite rays is formed. A pair of opposite rays forms a straight angle. Straight angles have a measure of 180 degrees. Therefore the sum of the three angles of a triangle must equal 180 degrees.

14. How might you adapt this activity for use with your students?
Have participants share their ideas with the entire group.
Some suggestions are:
- Do this activity with students at the beginning of the course, have students place their paper in a sheet protector in their notebook, then refer back to it as concepts are covered during the year.
- Have a poster size representation of the activity to refer to during the year.

Conjectures Activity 1: Triangles Tell it All

1. Construct a large scalene, acute triangle on an index card.

2. Cut out the triangle.

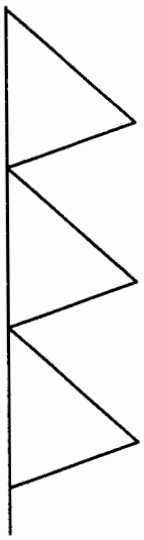
3. Verify that your triangle is scalene, acute.

4. Color each angle of the triangle a different color.

5. On a blank sheet of paper, construct a segment close to and parallel to the long edge. Make the segment as long as possible.

6. At the top of the segment match one edge of the triangle and trace the triangle.

7. Translate your triangle down the segment until the top vertex now rests on the bottom vertex. Trace the triangle. Color the angles the same as the original triangle.



8. Repeat step 7 until you run out of room. (You should have something that looks like saw teeth at right.)

9. Take the triangle and place it on top of the first traced triangle.

10. Rotate the triangle 180^0 about the midpoint of either of the 2 sides that are not on the original segment. Trace the triangle and color the angles the same as the original triangle.

11. Repeat step 10 on each triangle until you have filled the page.

12. What geometric relationships can you discover in this picture?

13. Select two relationships and justify/prove that the relationships are true. Explain your thinking.

14. How might you adapt this activity for use with your students?

Conjectures Activity 2: Midpoint Triangle

1. **On patty paper construct a large triangle.**
   Model as participants construct triangles. Note when using patty paper the presenter should discuss instructional techniques for patty paper. Keeping in mind the different learning styles of students, the teacher should model on the overhead and on a sheet of patty paper as well as giving verbal directions. Teachers should also circulate to help individuals understand directions and assist kinesthetic learners.

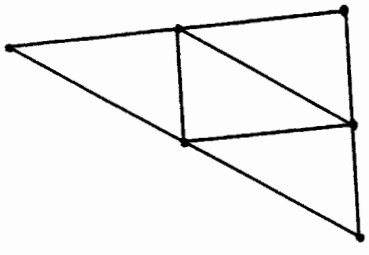
2. **Cut out the triangle.**
   Model as participants cut out their triangles.

3. **Classify your triangle according to its appearance. What type of triangle does it appear to be?**
   Encourage participants to classify using sides and angles for example, acute, scalene.

4. **Verify your classification. Explain your thinking.**
   Have participants share how they verified their classification. Methods might include: paper folding, tracing, measuring etc.

5. **Use paper folding to find the midpoint of each side of the triangle.**
   Model the "roll and pinch" method if necessary.

6. **Draw line segments connecting the midpoints like the sample below.**



7. **What figure did the midpoint segments form?**
   Triangle. *Types of triangles will vary depending on the original triangle. Ask participants how they verified the type of triangle.*

8. **Describe the relationships that can be found in the figure.**
   Have participants use paper folding or other methods to make and verify conjectures. Relationships may include:
   - *The segment that joins the midpoints of two sides of a triangle is parallel to and one-half the length of the third side.*
   - *The area of the new triangle is ¼ the area of the original triangle.*

9. **How can you justify/prove the relationships are true?**
   Ask for volunteers to share how they justified a relationship. One example might be cutting the figure into three triangles and stacking them to verify congruence.

10. **How might you adapt this activity for use with your students?**
    Ask for suggestions from participants. A possible suggestion might be to use the construction to focus on one or two of the relationships as appropriate.

Conjectures Activity 3: Midpoint Quadrilateral

1. **On patty paper construct a large convex quadrilateral.**
   Model as participants construct quadrilaterals. Note when using patty paper the presenter should discuss instructional techniques for patty paper. Keeping in mind the different learning styles of students the teacher should model on the overhead and on a sheet of patty paper as well as giving verbal directions. Teachers should also circulate to help individuals understand directions and assist kinesthetic learners.

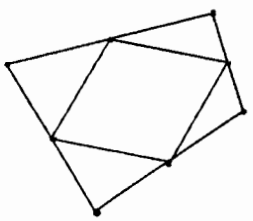
2. **Cut out the quadrilateral.**
   Model as participants cut out their quadrilaterals.

3. **Classify your quadrilateral according to its appearance. What type of quadrilateral does it appear to be?**
   Encourage participants to classify using sides and angles for example, rectangle, parallelogram, no special quadrilateral etc.

4. **Verify your classification. Explain your thinking.**
   Have participants share how they verified their classification. Paper folding, tracing, measuring etc.

5. **Find the midpoint of each side of the quadrilateral.**
   Model the "roll and pinch" method if necessary.

6. **Draw line segments connecting the midpoints like the sample below.**



7. **What figure did the midpoint segments form?**
Parallelogram. *Types of parallelograms will vary depending on the original quadrilateral. Ask participants how they verified the type of quadrilateral.*

Sample method: For diagram 1 we know that $\overline{EH} \parallel \overline{AC}$ *and* $\overline{FG} \parallel \overline{AC}$ *because the segment that joins the midpoints of two sides of a triangle is parallel to the third side. Since* $\overline{EH}$ *and* $\overline{FG}$ *are both parallel to* $\overline{AC}$ *then they are parallel to each other because if two lines are parallel to the same line then they are parallel to each other.*

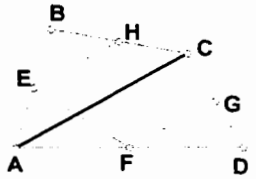


Diagram 1

For diagram 2 we know that $\overline{EF} \parallel \overline{BD}$ *and* $\overline{HG} \parallel \overline{BD}$ *because the segment that joins the midpoints of two sides of a triangle is parallel to the third side. Since* $\overline{HG}$ *and* $\overline{EF}$ *are both parallel to* $\overline{BD}$ *then they are parallel to each other because if two lines are parallel to the same line then they are parallel to each other.*

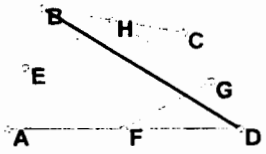


Diagram 2

Conclusion: Since both pairs of opposite sides of Quadrilateral ABCD are parallel ABCD must be a parallelogram.

8. **Describe the relationships that can be found in the figure.**
Have participants use paper folding or other methods to make and verify conjectures. *Relationships may include:*
   - The area of the center quadrilateral is ½ the area of the original quadrilateral.

9. **How can you justify/prove that the relationships are true?**
Ask for volunteers to share how they justified a relationship. One example might be cutting the figure into four triangles and the parallelogram, stacking them to verify area.

10. **How might you adapt this activity for use with your students?**
Answers will vary.

Activity 2: Midpoint Triangle

1. On patty paper construct a large triangle.

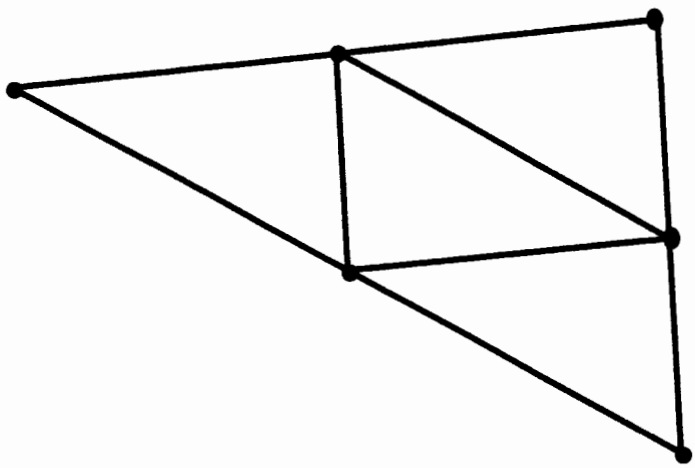
2. Cut out the triangle.

3. Classify your triangle according to its appearance. What type of triangle does it appear to be?

4. Verify your classification. Explain your thinking.

5. Use paper folding to find the midpoint of each side of the triangle.

6. Draw line segments connecting the midpoints like the sample below.



7. What figure did the midpoint segments form?

8. Describe the relationships that can be found in the figure.

9. How can you justify/prove the relationships are true?

10. How might you adapt this activity for use with your students?